

Bulk scalar field in warped extra dimensional models

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This work presents a general formalism to analyze a generic bulk scalar field in a multiple warped extra-dimensional model with arbitrary number of extra dimensions. The Kaluza-Klein mass modes along with the self-interaction couplings are determined and the possibility of having lowest lying KK mode masses near TeV scale are discussed. Also some numerical values for low-lying KK modes has been presented showing explicit localization around TeV scale. It is argued that the appearance of large number of closely spaced KK modes with enhanced coupling may prompt possible new signatures in collider physics.

I. INTRODUCTION

Theories with extra spacetime dimensions have drawn considerable attention ever since the original proposal by Kaluza and Klein. There has been renewed interest in such theories since the emergence of string theory. Several new ideas in this context have been proposed and have interesting consequences for particle phenomenology and cosmology [1–5]. In these higher-dimensional models, spacetime is usually taken to be a product of a four-dimensional spacetime and a compact manifold of dimension n . While gravity can propagate freely through the extra dimensions, Standard Model particles are confined to the four dimensional spacetime. Observers in this three spatial-dimensional wall (a “3-brane”) will measure an effective Planck scale $M_{pl}^2 = M^{n+2}V_n$, where V_n is the volume of the compact space. If V_n is large enough it could make Planck scale of the order of TeV, thus removing the hierarchy between the Planck and the weak scale.

Subsequently, Randall *et al.* [6, 7] proposed a higher-dimensional scenario that is based on nonfactorizable geometry and accounts for the hierarchy without introducing large extra dimensions. However, the braneworld model itself is not stable and it was shown in Ref. [8] that by introducing a scalar field in the bulk, the modulus-namely the brane separation in the RS model-can be stabilized without any fine-tuning. Assumption of negligibly small scalar backreaction on the metric in the GW approach prompted further work in this direction, where the modification of the RS metric due to backreaction of the bulk fields has been derived (see [9]). The stability issues in such cases have been reexamined for time-dependent cases [10, 11]; also the effect of gauge fields or higher form fields have been studied in several works (see [12]).

In an effort to search for the signatures of extra dimensions, the roles of the Kaluza-Klein modes of different bulk fields on the phenomenology at the standard model brane are of crucial importance.

For the five-dimensional RS model, [13] determined the bulk scalar KK modes and their self-interactions. It is found that due to the exponential redshift factor in the RS model, KK scalar modes in this spacetime have TeV scale mass splitting and inverse TeV couplings (see [7]). This is in sharp contrast to the KK decomposition in product spacetimes, which for large compactified dimensions, give rise to a large number of light KK modes (see [14]) with a very small coupling with brane fields. Due to this very distinct feature, the RS model has interesting consequences [13, 15].

Motivated by string theory and other extra-dimensional models where one can have several extra dimensions, in this paper we extend the results of the bulk scalar model in five-dimensional RS spacetime to arbitrary number of warped dimensions and have obtained the KK decomposition of the scalar KK masses. We have shown that in these multiply warped models we have much larger number of KK modes than the five-dimensional RS counterpart with effective couplings in the inverse TeV range. We have also discussed possible numerical values for various parameters in our theory and have used them to get possible numerical values of low-lying KK mode masses in our multiply warped model, showing

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explicit localization in TeV brane. Our results also establish a general formula for determining these KK masses and couplings in the presence of any arbitrary number of extra dimensions.

The paper is organized as follows: We give a brief explanation for six-dimensional doubly warped spacetime in Sec. II, the calculation for bulk scalar field has been done in this six-dimensional spacetime in Sec. III. The same calculations have been finally extended to higher-dimensional spacetime with arbitrary number of extra dimensions in Secs. IV and V. The paper ends with a short discussion of our results.

II. SIX-DIMENSIONAL DOUBLY WARPED SPACETIME AND EINSTEIN EQUATIONS

In this section we shall discuss doubly compactified six-dimensional spacetime with Z_2 orbifolding along each compactified direction. For a detailed discussion we refer the reader to [15]. The manifold under consideration is given by, $M^{1,5} = [M^{1,3} \times S^1/Z_2] \times S^1/Z_2$ [15].

We let the compactified dimensions to y and z , respectively. The noncompactified dimensions are taken to be, x^μ ($\mu = 0, 1, 2, 3$). The moduli along the compact dimensions are given by R_y and r_z , respectively. The corresponding metric ansatz is taken as

$$ds^2 = b^2(z) [a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2, \quad (1)$$

with $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. Thus we have four orbifold fixed points, which are given by $(y, z) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$, respectively.

The total bulk-brane action could be given by

$$S = S_6 + S_5 + S_4 \quad (2)$$

$$S_6 = \int d^4x dy dz \sqrt{-g_6} (R_6 - \Lambda_6) \quad (3)$$

$$S_5 = \int d^4x dy dz [V_1 \delta(y) + V_2 \delta(y - \pi)] + \int d^4x dy dz [V_3 \delta(z) + V_4 \delta(z - \pi)] \quad (4)$$

$$S_4 = \sum_{i=1}^2 \sum_{j=1}^2 \int d^4x dy dz \sqrt{-g_{vis}} (L - V) \delta(y - y_i) \delta(z - z_j). \quad (5)$$

Here the brane potentials in general have the particular functional dependence $V_{1,2} = V_{1,2}(z)$ and $V_{3,4} = V_{3,4}(y)$. Finally the full six-dimensional Einstein's equation is given by,

$$\begin{aligned} -M^4 \sqrt{-g_6} \left(R_{MN} - \frac{R}{2} g_{MN} \right) = & \Lambda_6 \sqrt{-g_6} g_{MN} \\ & + \sqrt{-g_5} V_1(z) g_{\alpha\beta} \delta_M^\alpha \delta_N^\beta \delta(y) \\ & + \sqrt{-g_5} V_2(z) g_{\alpha\beta} \delta_M^\alpha \delta_N^\beta \delta(y - \pi) \\ & + \sqrt{-\tilde{g}_5} V_3(y) \tilde{g}_{\tilde{\alpha}\tilde{\beta}} \delta_M^{\tilde{\alpha}} \delta_N^{\tilde{\beta}} \delta(z) \\ & + \sqrt{-\tilde{g}_5} V_4(y) \tilde{g}_{\tilde{\alpha}\tilde{\beta}} \delta_M^{\tilde{\alpha}} \delta_N^{\tilde{\beta}} \delta(z - \pi) \end{aligned} \quad (6)$$

In the above expression M, N are bulk indices, α, β run over the usual four spacetime coordinates given by x^μ . The quantities g and \tilde{g} are the metric in $y = \text{extremconstant}$ and $z = \text{constant}$ hypersurfaces, respectively. The line element derived from the above Einstein's equation turns out to be [15]

$$ds^2 = \frac{\cosh^2(kz)}{\cosh^2(k\pi)} [\exp(-2c|y|)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2. \quad (7)$$

In the above line element we have the following identification for the constants k and c given by

$$\begin{aligned} c &\equiv \frac{R_y k}{r_z \cosh(k\pi)} \\ k &\equiv r_z \sqrt{\frac{-\Lambda_6}{10M^4}}. \end{aligned} \quad (8)$$

The boundary terms lead to the brane tensions and using the Einstein's equation across the two boundaries at $y = 0$, $y = \pi$, respectively, thus we readily obtain

$$V_1(z) = -V_2(z) = 8M^2 \sqrt{\frac{-\Lambda_6}{10}} \text{sech}(kz). \quad (9)$$

Thus the two 4-branes situated at $y = 0$ and $y = \pi$ would have a z -dependent brane tension. The fact that the two tensions are equal and opposite is reminiscent of the original RS-form. Similarly we get the brane tensions for other two 4-branes as

$$V_3(y) = 0; V_4(y) = -\frac{8M^4 k}{r_z} \tanh(k\pi). \quad (10)$$

Here $V_{3,4}$ were introduced to account orbifolding along the z -direction and with g_{zz} being a constant, the resulting hypersurface should have only a constant energy density. The fact that g_{yy} is dependent on the coordinate z makes the two hypersurfaces for y orbifolding to have a z -dependent energy density.

The 3-brane located at $(y = 0, z = \pi)$ suffers no warping and can be identified with the Planck brane. The other three can be valid choices for Standard Model (visible) brane. However, if we assume that there is no brane having lower energy scale than ours, we are forced to identify the SM brane to be located at $(y = \pi, z = 0)$. The suppression factor on the TeV brane can be given by

$$f = \frac{\exp(-c\pi)}{\cosh(k\pi)}. \quad (11)$$

The desired suppression of 10^{-16} on the TeV brane can be obtained by choosing different combinations of the parameters c and k . However we also have an extra relation as presented in Eq. (8), which shows that in order to avoid large hierarchy between the two moduli R_y and r_z , either of the two parameters c and k must be large and the other should be small. For example, we can easily assume $c \sim 11.4$ and $k \sim 0.1$. However a small hierarchy also exists in the original RS model, where there exists an one order of magnitude hierarchy between r and k , satisfying Planck-to-TeV scale warping by $kr \sim 11.5$. A natural question that arises with this discussion is whether stabilization of these moduli to the desired values is possible. For the five-dimensional RS model this has been shown in [8] by introducing a bulk stabilizing scalar field and tuning the VEV of the scalar field at the boundaries. The modulus in the theory is stabilized near Planck length without any fine-tuning.

In this case as well we can adopt a similar procedure by introducing a bulk stabilizing scalar field. Again choosing appropriate VEV at the boundary, we can stabilize R_y and r_z to desired values [16]. In our six-dimensional braneworld scenario with y and z dependence, the action for the scalar field can be expressed such that

$$S = \int d^4x dy dz \sqrt{-g_6} \left(\frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right) + \sum_{i,j=1}^2 \int d^4x \sqrt{-g_{ij}} \lambda_{ij}(\phi) (\phi^2 - v_{ij}^2)^2 \delta(y - y_i) \delta(z - z_j), \quad (12)$$

where the coupling parameters, λ_{ij} tend to infinity as the scalar field approach to the following values, $\phi(0,0) = v_0$, $\phi(0,\pi) = v_1$, $\phi(\pi,0) = v_3$ and $\phi(\pi,\pi) = v_4$. We take $V(\phi) = m^2 \phi^2$. Now following Ref. [8, 17] we can obtain the equation of motion in the separable form as

$$\psi''(y) - 4c\psi'(y) = p\psi(y) \\ \frac{b^2 R_y^2}{r_z^2} \left[\ddot{\chi}(z) + \frac{5\dot{b}}{b} \dot{\chi}(z) \right] = (R_y^2 b^2 m^2 - p) \chi(z), \quad (13)$$

where we have made the following decomposition, $\phi(y,z) = \psi(y)\chi(z)$ and p is the separability constant. Also in the above expression prime denote differentiation with respect to y and dot denotes differentiation with respect to z . Finally, the above equations with appropriate boundary condition [17] can be solved, which when substituted into the action leads to an effective potential for the moduli as

$$V_{eff} = \pi v_2^2 \left[\frac{(1 - 2v + v^2)}{2k\nu\pi} + \frac{k}{12\nu} \left((1 + 2\alpha - 8\nu + 2\nu^2) + v(22 + 2\alpha - 8\nu + 2\nu^2) + (1 + 2\alpha - 8\nu + 2\nu^2) \right) \right], \quad (14)$$

where we have used the following shorthand notations, $v = v_1/v_2$, $\alpha = -10m^2M^4/\Lambda_6$ and $\nu = \sqrt{4 + \frac{p}{c^2}}$. Then solving the equations, $\partial_\nu V_{eff} = 0$ and $\partial_k V_{eff} = 0$ and then through the second derivatives with respect to ν and k we can arrive at the minimum values of c and k . As an illustrative example we can start with $v = 0.43$ and $p \sim 1$, leading to $c \sim 11.24$ and $k \sim 0.422$. Note that these values can resolve the gauge hierarchy problem. Thus along this line any higher-dimensional braneworld models can have their moduli stabilized. From now on we shall assume that such a stabilization has been performed and all the moduli hence forth will have those stabilized values. In this analysis, following the stabilizing bulk scalar model, we have assumed that the backreaction of the bulk stabilizing field is negligibly small. Moreover from the action of the bulk stabilizing scalar it may be observed that at the boundaries the stabilizing scalar tends to their VEVs v_{ij} when the coupling $\lambda_{ij}(\phi)$ tends to infinity. This is exactly similar to the five-dimensional counter part of the Goldberger-Wise calculation of modulus stabilization. As a result at the boundaries, the stabilizing scalar is frozen to different values v_{ij} and hence does not contribute to the dynamics of the model.

III. BULK FIELD IN SIX-DIMENSIONAL DOUBLY WARPED SPACETIME

In this section we carry out the Kaluza-Klein decomposition of a nongravitational bulk scalar field propagating in the spacetime described by Eq. (7) in the spirit of the work [13] with bulk scalar field. We find that in these multiply warped spacetime the SM brane contains larger number of TeV scale scalar KK modes than the five-dimensional RS model. This has significant phenomenological consequences [18]. We consider a free scalar field in the bulk for which the action is given by

$$S = \frac{1}{2} \int d^4x \int dy \int dz \sqrt{-G} (G^{AB} \partial_A \Phi \partial_B \Phi + m^2 \Phi^2), \quad (15)$$

where G_{AB} with $A, B = \mu, y, z$ is given by Eq. (7), and m is of order of M_{pl} . After an integration by parts, this can be written as

$$\begin{aligned} S = & \frac{1}{2} \int d^4x \int dy \int dz \left[R_y r_z e^{-2\sigma} \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \eta_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right. \\ & + R_y r_z e^{-4\sigma} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} m^2 \Phi^2 \\ & - \frac{r_z}{R_y} \Phi \partial_y \left(e^{-4\sigma} \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \partial_y \Phi \right) \\ & \left. - \frac{R_y}{r_z} \Phi \partial_z \left(e^{-4\sigma} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \partial_z \Phi \right) \right], \end{aligned} \quad (16)$$

where $\sigma = c|y|$. Now we make the following substitution for KK decomposition,

$$\Phi(x, y, z) = \sum_{n,m} \phi_{nm}(x) \frac{\alpha_n(y)}{\sqrt{R_y}} \frac{\beta_m(z)}{\sqrt{r_z}}. \quad (17)$$

The following normalization conditions are imposed on the fields α and β ,

$$\int dy e^{-2\sigma} \alpha_n \alpha_m = \delta_{nm} \quad (18)$$

$$\int dz \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \beta_p \beta_q = \delta_{pq}. \quad (19)$$

The differential equation satisfied by the function $\alpha_n(y)$ is

$$- \frac{1}{R_y^2} \frac{d}{dy} \left(e^{-4\sigma} \frac{d\alpha_m}{dy} \right) = A_m^2 e^{-2\sigma} \alpha_m, \quad (20)$$

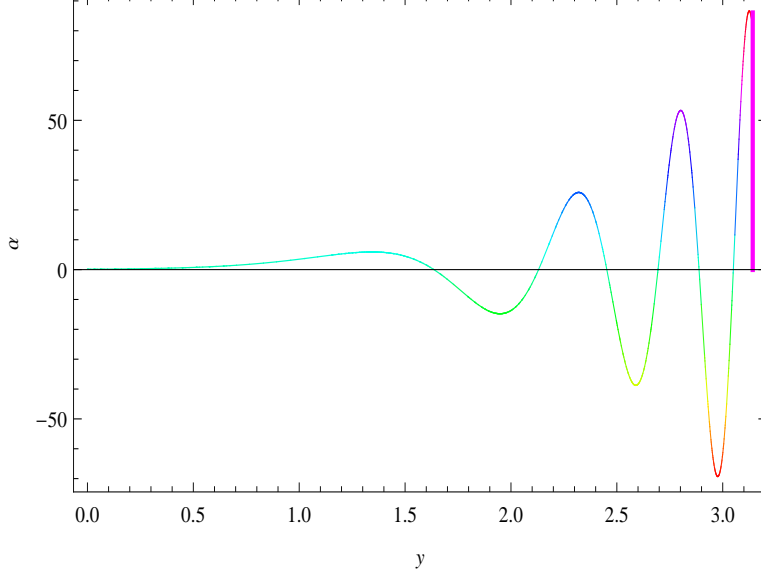


FIG. 1: The figure shows variation of the quantity α_m with extra-dimension parameter y . The vertical line represents the $y = \pi$ line showing the fact that the quantity α_m is maximum at $y = \pi$, the position of TeV brane.

where A_m stands for KK mode mass eigenvalue. The above differential equation can be further simplified and cast to the following form,

$$\frac{d^2 \alpha_m}{dy^2} - 4c \frac{d\alpha_m}{dy} + A_m^2 R_y^2 e^{2\sigma} \alpha_m = 0. \quad (21)$$

The above equation can be solved in terms of Bessel functions of first and second order as

$$\alpha_m = \frac{e^{2\sigma}}{N_m} \left[J_2 \left(\frac{A_m e^\sigma R_y}{c} \right) + b_m Y_2 \left(\frac{A_m e^\sigma R_y}{c} \right) \right], \quad (22)$$

with N_m representing an overall normalization. Now we can proceed much further. The mass modes determined by A_m must be real. This reality condition imposed on the mass modes requires the differential operator on the left hand side of Eq. (22) to be self-adjoint. This self-adjointness imply that derivatives of $\alpha_m(y)$ should be continuous at the orbifold fixed points. These gives two conditions on the parameters A_m and b_m , expressed as,

$$b_m = - \frac{2J_2 \left(\frac{A_m R_y}{c} \right) + \frac{A_m R_y}{c} J_2' \left(\frac{A_m R_y}{c} \right)}{2Y_2 \left(\frac{A_m R_y}{c} \right) + \frac{A_m R_y}{c} Y_2' \left(\frac{A_m R_y}{c} \right)} \quad (23)$$

$$0 = [2J_2(x_m) + x_m J_2'(x_m)] [2Y_2(x_m e^{-c\pi}) + x_m e^{-c\pi} Y_2'(x_m e^{-c\pi})] - [2Y_2(x_m) + x_m Y_2'(x_m)] [2J_2(x_m e^{-c\pi}) + x_m e^{-c\pi} J_2'(x_m e^{-c\pi})] \quad (24)$$

where $x_m = A_m e^{c\pi} R_y / c$. Since to make Planck scale down to TeV scale we should have $e^{c\pi} \gg 1$. Then the above equation reduces to the following form, $2J_2(x_m) + x_m J_2'(x_m) = 0$. Then for light mode masses we have x_1 to be order of unity [13]. This keeps $A_m R_y / c$ also order of unity. Then from Eq. (22) as well as from Fig.1 we observe that the modes $\alpha_m(y)$ are larger near the 3-brane at $y = \pi$, which makes these low-mass Kaluza-Klein modes to be found preferentially near the $y = \pi$ region (see Fig.1). Thus, with the TeV brane being situated at $y = \pi$, we observe that the low-mass KK modes are exponentially suppressed and hence confined to the TeV brane. Also for these low-lying KK mass modes the coefficient b_m is of the order of $e^{-4c\pi}$, which shows that we can ignore the $Y_2(y_m)$ part compared to $J_2(y_m)$, while performing integrals involving α_m .

Similar analysis for β_m yields

$$\frac{d^2 \beta_m}{dz^2} + 5k \tanh(kz) \frac{d\beta_m}{dz} + r_z^2 B_m^2 \frac{\cosh^2(k\pi)}{\cosh^2(kz)} \beta_m = 0. \quad (25)$$

The solution, apart from an overall normalization, can be expressed as

$$\begin{aligned} \beta_m(z) = & \exp\left[-\frac{5}{2}k^2z^2\right] H_{\sqrt{5/2}kz} \left(\frac{-10k^2 + B_m^2 r_z^2(1 + \cosh(2k\pi))}{10k^2}\right) \\ & + E_m \exp\left[-\frac{5}{2}k^2z^2\right] {}_1F_1\left(-\frac{-10k^2 + B_m^2 r_z^2(1 + \cosh(2k\pi))}{10k^2}, \frac{1}{2}, \frac{5k^2z^2}{2}\right), \end{aligned} \quad (26)$$

where ${}_1F_1$ is the Kummer confluent hypergeometric function and H_n is the Hermite polynomial of degree n . Then from Fig.2 we observe that this function is also maximum at $z = 0$. Hence the low-lying KK mass modes are confined to the TeV brane located at $z = 0$. From the behavior of both α_m and β_n we find that all the low-lying KK mass modes are confined to $y = \pi, z = 0$ brane, i.e., the TeV brane. A possible experimental signature of the bulk scalar KK modes can originate via coupling of the bulk scalar to diHiggs in the form $\Phi(x)h^2(x)$. For $m_\Phi \sim m_h$ the dominant decay channels are gg and $b\bar{b}$ which leads to multijets as final product which though may be difficult to differentiate from the QCD back ground [19–21]. Also when the mass of bulk scalar is in the range of 250 to 350 GeV then enhanced production of $\Phi \rightarrow hh$ occurs. Moreover for bulk scalar mass in the range 160 to 250 GeV we have a relatively larger cross sections for the diphoton channel. In this region due to small mixing the branching ratio is dominated by gg and $b\bar{b}$. The diphoton channel is a very promising search channel as branching ratio remains more or less at constant level even up to $t\bar{t}$ threshold [22, 23]. This might become possible if the LHC runs extends the diphoton searches for invariant masses above existing $m_{\gamma\gamma} = 150\text{GeV}$.

To determine the parameters of the solution we proceed as follows: we want B_m to be real, as it appears in the mass modes. Thus self-adjointness also applies in this case. This implies that derivatives of β_m to be continuous around the orbifold fixed points. At $z = 0$, this is trivially satisfied irrespective of the quantity E_m . However at $z = \pi$ all the terms are suppressed by $\exp(-5k^2z^2/2)$, thus the self-adjointness there leads to

$$E_m = -\frac{H_{\sqrt{5/2}k\pi}(a)}{{}_1F_1\left(-a, \frac{1}{2}, \frac{5k^2z^2}{2}\right) + 2a {}_1F_1\left(a + 1, \frac{3}{2}, -\frac{5}{2}k^2z^2\right)} \quad (27)$$

$$a = \frac{-10k^2 + B_m^2 r_z^2(1 + \cosh(2k\pi))}{10k^2}. \quad (28)$$

At large values of z , confluent hypergeometric series have a large value. Being in the denominator the term can be neglected for practical purposes. Using the above equations we readily obtain the following action for the field $\phi(x)$ as,

$$S = \frac{1}{2} \int d^4x \left(\sum_{n,m} \eta^{\mu\nu} \partial_\mu \phi_{nm} \partial_\nu \phi_{nm} + \sum_{n,m,p,q} M_{nmpq} \phi_{nm} \phi_{pq} \right) \quad (29)$$

$$M_{nmpq} = \{A_n^2 \delta_{np} \delta_{mq} + B_n^2 \delta_{np} P_{mq} + m^2 P_{np} Q_{mq}\} \quad (30)$$

where we have the following expression for the element Q_{nm} ,

$$Q_{nm} = \int dz \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \beta_n \beta_m \quad (31)$$

and P_{mn} as,

$$P_{nm} = \int dy e^{-4\sigma} \alpha_n \alpha_m \quad (32)$$

Now from the previous discussion we have the solution for these two sets of functions $\alpha_n(y)$ and $\beta_n(z)$, which can be used to determine Q_{nm} and P_{nm} in order to obtain the masses of the KK modes by evaluating the quantity M_{nmpq} . In contrast to the five-dimensional situation (see [13]) where the masses of the bulk fields appear as a diagonalized mass matrix, in this case the bulk field $\Phi(x, y, z)$ manifests itself to some four-dimensional observer as an infinite KK tower with mass being determined by the the quantity M_{nmpq} such that a scalar ϕ_{nm} has a mass M_{nmnm} after an appropriate diagonalization procedure.

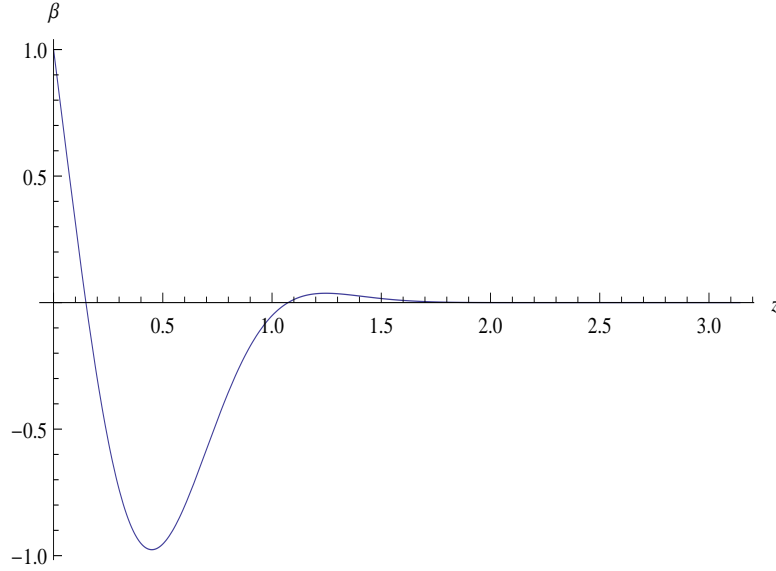


FIG. 2: The figure shows variation of the quantity β_n with extra-dimension parameter z . The graph clearly shows the fact that the quantity β_n is maximum at $z = 0$, the position of TeV brane.

The solution for $\alpha_n(y)$ is presented in equation (22). A similar solution was obtained by Wise et. al (see [13]) except for the fact that we have Bessel functions of second order. Following their discussion we can argue in a similar manner that the lightest KK modes have mass parameter A_m suppressed exponentially with respect to the scale m appearing in Eq. (15). Since we have taken m to be order of Planck scale and c to be around 12, by stabilization these mass modes A_m are in the TeV range. Also from the solution we could observe that the modes $\alpha_n(y)$ are larger in the region $y = \pi$. This has been explained earlier through graphical presentation of the function α_m .

The solution for $\beta_n(z)$ has been presented in Eq. (26). Though we have argued following the graphical presentation of the function β_n , we now provide a theoretical motivation for our above-mentioned results. The solution has an overall factor of $\exp[-\frac{5}{2}k^2z^2]$ and we see that the solution has maximum value around $z = 0$. Hence the bulk field being a product of these two functions $\alpha_m(y)$ and $\beta_n(z)$ as shown in Eq. (16) has mass parameter in the TeV range and has maximum value around $(y = \pi, z = 0)$. Now from Sec. II this is precisely the SM brane. Hence, the bulk field has a maximum in the SM brane; i.e., the KK modes are most likely to be found in that region where A_m and B_m are in the TeV range. This sets the stage for KK excitations to have TeV scale mass splitting on the SM brane.

Now we would like to compute some low-lying KK mode masses numerically. For that we need to fix some parameters, k , c , r_z and the bulk mass of the scalar field m . We shall take the bulk mass to be in Planck scale. Then we can determine the remaining parameters, by making the following demands: (a) if we have a gauge boson field in this multiply warped scenario, its lowest massive KK modes should lead to W and Z boson masses, (b) the suppression f as presented in Eq. (11) should be $\sim 10^{-16}$, and finally (c) the hierarchy between R_y and r_z should be small. The KK mode of the gauge boson in this multiply warped spacetime can be obtained from Ref. [24].

This desired mass for W and Z boson $\sim 100\text{GeV}$ can be obtained with $f \sim 10^{-16}$ and $\frac{1}{r_z} = 7 \times 10^{17}\text{GeV}$, about 14 times smaller compared to Planck scale. The other parameters k and c can be determined using small hierarchy between R_y and r_z along with desired warping of $f \sim 10^{-16}$. This finally leads to, the following estimation: $k = 0.25$, $c = 11.52$ and the ratio between moduli being $\frac{R_y}{r_z} = 61$. The suppression factor turns out to be $f = 1.45 \times 10^{-16}$. Thus we will calculate the low-lying KK masses for our bulk scalar field with these sets of parameters (see Table.I).

We now present the self-interactions of the bulk scalar field. From the four-dimensional point of view these self-interactions can induce couplings between the KK modes. In this case self-couplings of the light modes are suppressed by the warp factor and hence if the Planck scale set the six-dimensional couplings, the low-lying KK modes have TeV range self-interactions. We present the interaction term in the action

TABLE I: The masses of the KK modes of the scalar field are given in GeV units. We have chosen the following values, $\frac{1}{r_z} = 7 \times 10^{17}$ GeV, $k = 0.25$, $c = 11.52$. Some representative masses of low-lying KK modes are given.

$m_{1111} = 99.513$	$m_{1212} = 99.651$	$m_{1313} = 99.709$	$m_{1414} = 99.743$
$m_{2121} = 178.614$	$m_{2222} = 178.866$	$m_{2323} = 178.965$	$m_{2424} = 179.026$
$m_{3131} = 257.714$	$m_{3232} = 258.069$	$m_{3333} = 258.228$	$m_{3434} = 258.309$
$m_{4444} = 337.592$	$m_{5555} = 416.957$	$m_{6666} = 501.445$	$m_{7777} = 583.371$

with coupling parameter λ such that

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \sqrt{G} \frac{\lambda}{M^{4m-6}} \Phi^{2m}, \quad (33)$$

where the coupling λ is of the order of unity. Then we can expand in modes and the self-interactions of light KK states become

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz R_y r_z e^{-4\sigma} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \frac{\lambda}{M^{4m-6}} \phi_{pq}^{2m} \left(\frac{\alpha_p}{\sqrt{R_y}} \frac{\beta_q}{\sqrt{r_z}} \right)^{2m}. \quad (34)$$

Thus the effective four dimensional coupling constants are

$$\lambda_{eff} = \frac{4\lambda}{(MR_y)^{m-1} (Mr_z)^{m-1} M^{2m-4}} \int_0^{\pi} dy e^{-4\sigma} \alpha_p^{2m} \int_0^{\pi} dz \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \beta_q^{2m}, \quad (35)$$

which reduces to,

$$\lambda_{eff} \simeq 4\lambda \left(\frac{c}{MR_y} \right)^{m-1} \left(\frac{1}{Mr_z} \right)^{m-1} \left(M e^{-c\pi} \frac{1}{\cosh(k\pi)} \right)^{4-2m} \int_0^1 r^{4m-5} dr \left[\frac{J_2 \left(\frac{A_p e^{\sigma}}{k} r \right)}{A_p} \right]^{2m} \int_0^{\pi} (\beta_q)^{2m} dz \quad (36)$$

in the large kR_y and kr_z limit. Hence we observe that the relevant scale for four-dimensional physics is not the scale set by Planck scale, i.e., M , but this is $M e^{-c\pi} \frac{1}{\cosh k\pi}$. Hence the KK reduction has lead the couplings from Planck scale to the TeV scale by the warp factor on the SM brane located at ($y = \pi, z = 0$).

From the above discussion we now try to obtain some bounds on the parameters in our model, e.g., R_y , r_z from the requirement of precision electroweak test. For that purpose we can use the same setup and put a bulk gauge boson whose KK modes can be detected in precision electroweak tests. We define a quantity denoted by

$$V = \sum_{n=1}^{\infty} \left(\frac{g_n^2}{g_0^2} \frac{M_W^2}{M_n^2} \right), \quad (37)$$

where M_W is the mass of W gauge boson and M_n is the mass of higher KK modes of the bulk gauge boson and g_0 is the effective four-dimensional gauge coupling along with g_n to be the gauge couplings for higher KK modes. Then from Ref. [25] we could argue that for precision electroweak test we should have $V < 0.0013$ with 95 percent confidence level. From this result we can get the following bounds on the parameters of this model, $1/R_y < 5.95 \times 10^{17}$ GeV. This leads to a bound on $1/r_z$ as well by assuming a small hierarchy between the two moduli as, $1/r_z < 3.63 \times 10^{19}$ GeV. From Ref. [24] it can be easily verified that this bound is respected by gauge couplings and KK mode masses. Thus these multiply warped models indeed satisfy precision electroweak tests.

IV. SEVEN-AND-HIGHER-DIMENSIONAL SPACETIME WITH MULTIPLE WARPING

With an aim to arrive at a generic result we shall now try to extend our analysis with one more extra dimension. For that purpose we start with a seven-dimensional spacetime where the space-like dimensions are successively warped. In other words the manifold of interest could be given by

$[\{M^{(1,3)} \times [S^1/Z_2]\} \times S^1/Z_2] \times S^1/Z_2$. Then the total brane-bulk action can be given by

$$S = S_7 + S_6 + S_5 + S_4 \quad (38)$$

$$S_7 = \int d^4x dy dz dw \sqrt{-g_7} (R_7 - \Lambda_7) \quad (39)$$

$$\begin{aligned} S_6 = & \int d^4x dy dz dw [V_1 \delta(w) + V_2 \delta(w - \pi)] \\ & + \int d^4x dy dz dw [V_3 \delta(z) + V_4 \delta(z - \pi)] \\ & + \int d^4x dy dz dw [V_5 \delta(y) + V_6 \delta(y - \pi)], \end{aligned} \quad (40)$$

with appropriate actions (S_5) for 12 possible 4-branes at the edges $(z, w) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$, $(z, y) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$ and $(y, w) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$. We also have eight possible 3-branes at the corners $(y, z, w) = (0, 0, 0), (0, 0, \pi), (0, \pi, 0), (\pi, 0, 0), (\pi, \pi, 0), (0, \pi, \pi), (\pi, 0, \pi), (\pi, \pi, \pi)$. By natural extension of the method as illustrated in the previous section we get the line element and other parameters such that [15],

$$\begin{aligned} ds^2 &= \frac{\cosh^2(\ell w)}{\cosh^2(\ell \pi)} \left\{ \frac{\cosh^2(kz)}{\cosh^2 k \pi} [exp(-2c|y|) \eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2 \right\} + \mathfrak{R}_w^2 dw^2 \\ \ell^2 &= \frac{-\Lambda_7 \mathfrak{R}_w^2}{15} \\ k &= \frac{\ell r_z}{\mathfrak{R}_w \cosh(\ell \pi)} \\ c &= \frac{\ell R_y}{\mathfrak{R}_w \cosh(k \pi) \cosh(\ell \pi)} = \frac{k R_y}{r_z \cosh(k \pi)} \end{aligned} \quad (41)$$

It may be of interest that the 5-brane at $w = \pi$ does not represent a flat metric (y and z dependencies). In order to obtain substantial warping along the w direction (from $w = \pi$ to $w = 0$), one need to make $\ell \pi$ substantial (same order of magnitude as RS scenario). The seven-dimensional or triply warped model has a structure analogous to that of six-dimensional one, not only in the, form of functional dependence but also on the nature of warping. This method can easily be extended to even higher dimensions. Also note that the orbifolding requires branes situated at edges of n-dimensional hypercube with 3-branes at the corners. If one of the direction suffers a large warping any other direction should have small warping so that there is no large hierarchy coming from the moduli. In this case also we have several candidates for our SM brane. However applying the fact that no brane should have less energy than ours, leads to $(y = \pi, z = 0, w = 0)$ to be SM brane.

V. BULK FIELDS IN SEVEN-AND-HIGHER-DIMENSIONAL SPACETIME

Following the methods of previous sections, we shall carry out the Kaluza-Klein decomposition of a bulk scalar field propagating in the spacetime given by Eq. (41). As in the previous section in this case as well we can write the bulk scalar field in terms of product of four functions. By making KK decomposition we again end up with KK mass modes having TeV scale masses and splittings. The action for the bulk scalar field in this seven-dimensional spacetime can be given as

$$S = \frac{1}{2} \int d^4x \int dy \int dz \int dw \sqrt{-G} [G_{AB} \partial^A \Phi \partial^B \Phi + m^2 \Phi^2]. \quad (42)$$

From the line element as given by Eq. (38), we readily obtain the following form for the action

$$\begin{aligned} S = & \frac{1}{2} \int d^4x \int dy \int dz \int dw \left[R_y r_z \mathfrak{R}_w e^{-2\sigma} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \frac{\cosh^3(kz)}{\cosh^3(k \pi)} \eta_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right. \\ & \left. + \frac{1}{2} \frac{\mathfrak{R}_w r_z}{R_y} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \frac{\cosh^3(kz)}{\cosh^3(k \pi)} e^{-4\sigma} (\partial_y \Phi)^2 \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{\Re_w R_y}{r_z} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} e^{-4\sigma} (\partial_z \Phi)^2 \\
& + \frac{1}{2} \frac{r_z R_y}{\Re_w} \frac{\cosh^6(\ell w)}{\cosh^6(\ell \pi)} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} e^{-4\sigma} (\partial_w \Phi)^2 \\
& + \frac{1}{2} m^2 \Re_w r_z R_y \frac{\cosh^6(\ell w)}{\cosh^6(\ell \pi)} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} e^{-4\sigma} \Phi^2,
\end{aligned} \tag{43}$$

where $\sigma = c|y|$. We make the following substitution for the bulk field:

$$\Phi = \sum_{pqr} \phi_{pqr}(x) \frac{\alpha_p}{\sqrt{R_y}} \frac{\beta_q}{\sqrt{r_z}} \frac{\gamma_r}{\sqrt{\Re_w}} \tag{44}$$

We also impose the following normalization for the functions α_p , β_q and γ_r ,

$$\int e^{-2} \alpha_m \alpha_n dy = \delta_{mn} \tag{45}$$

$$\int \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \beta_m \beta_n dz = \delta_{mn} \tag{46}$$

$$\int \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \gamma_m \gamma_n dw = \delta_{mn}. \tag{47}$$

Now applying integration by parts to the integral as presented in Eq. (43) we readily obtain

$$\begin{aligned}
S = & \frac{1}{2} \int d^4x \int dy \int dz \int dw \left\{ \left[\sum_{pqrabc} e^{-2\sigma} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \frac{\cosh^3(kz)}{\cosh^3(k\pi)} (\eta^{\mu\nu} \partial_\mu \phi_{pqr} \partial_\nu \phi_{abc}) \alpha_p \alpha_a \beta_q \beta_b \gamma_r \gamma_c \right] \right. \\
& - \frac{1}{2} \frac{1}{R_y^2} \left[\sum_{pqrabc} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \phi_{pqr} \phi_{abc} \beta_q \beta_b \gamma_r \gamma_c \alpha_p \partial_y (e^{-4\sigma} \partial_y \alpha_a) \right] \\
& - \frac{1}{2} \frac{1}{r_z^2} \left[\sum_{pqrabc} \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} e^{-4\sigma} \phi_{pqr} \phi_{abc} \alpha_p \alpha_a \gamma_r \gamma_c \beta_q \partial_z \left(\frac{\cosh^5(kz)}{\cosh^5(k\pi)} \partial_z \beta_b \right) \right] \\
& - \frac{1}{2} \frac{1}{\Re_w^2} \left[\sum_{pqrabc} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} e^{-4\sigma} \phi_{pqr} \phi_{abc} \alpha_p \alpha_a \beta_q \beta_b \gamma_r \partial_w \left(\frac{\cosh^6(\ell w)}{\cosh^6(\ell \pi)} \partial_w \gamma_c \right) \right] \\
& \left. + \frac{1}{2} m^2 \left[\sum_{pqrabc} \frac{\cosh^6(\ell w)}{\cosh^6(\ell \pi)} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} e^{-4\sigma} \phi_{pqr} \phi_{abc} \alpha_p \alpha_a \beta_q \beta_b \gamma_r \gamma_c \right] \right\}.
\end{aligned} \tag{48}$$

Then we make the following choice for the differential equations satisfied by the functions α_n , β_n and γ_n ,

$$- \frac{1}{R_y^2} \partial_y (e^{-4\sigma} \partial_y \alpha_n) = A_n^2 e^{-2\sigma} \alpha_n \tag{49}$$

$$- \frac{1}{r_z^2} \partial_z \left(\frac{\cosh^5(kz)}{\cosh^5(k\pi)} \partial_z \beta_n \right) = B_n^2 \frac{\cosh^3(kz)}{\cosh^3(k\pi)} \beta_n \tag{50}$$

$$- \frac{1}{\Re_w^2} \partial_w \left(\frac{\cosh^6(\ell w)}{\cosh^6(\ell \pi)} \partial_w \gamma_n \right) = C_n^2 \frac{\cosh^4(\ell w)}{\cosh^4(\ell \pi)} \gamma_n. \tag{51}$$

The first equation as presented in (49) can be solved and has an identical solution as that obtained in the previous section. However, for convenience we rewrite the solution,

$$\alpha_p = \frac{e^{2\sigma}}{N_p} \left[J_2 \left(\frac{A_p e^\sigma R_y}{c} \right) + b_p Y_2 \left(\frac{A_p e^\sigma R_y}{c} \right) \right] \tag{52}$$

The second equation as given by Eq. (50) has the same solution as presented in Eq. (26) but we rewrite it here,

$$\begin{aligned}\beta_q(z) = & \exp\left[-\frac{5}{2}k^2z^2\right] H_{\sqrt{5/2}kz} \left(\frac{-10k^2 + B_q^2 r_z^2(1 + \cosh(2k\pi))}{10k^2}\right) \\ & + E_q \exp\left[-\frac{5}{2}k^2z^2\right] {}_1F_1\left(-\frac{-10k^2 + B_q^2 r_z^2(1 + \cosh(2k\pi))}{10k^2}, \frac{1}{2}, \frac{5k^2z^2}{2}\right)\end{aligned}\quad (53)$$

The third Eq. (51) has the following solution along with an overall normalization,

$$\begin{aligned}\gamma_r(w) = & \exp[-3\ell^2w^2] H_{\sqrt{3}\ell w} \left(\frac{-12\ell^2 + C_r^2 \Re_w^2(1 + \cosh(2\ell\pi))}{12\ell^2}\right) \\ & + F_r \exp[-3\ell^2w^2] {}_1F_1\left(-\frac{-12\ell^2 + C_r^2 \Re_w^2(1 + \cosh(2\ell\pi))}{24\ell^2}, \frac{1}{2}, 3\ell^2w^2\right)\end{aligned}\quad (54)$$

Here also we have J_2 and Y_2 to be Bessel functions of first and second order respectively. Along with these H_n represents Hermite polynomials and ${}_1F_1$ is the Kummer confluent hypergeometric series. The arbitrary constants b_p , E_q and F_r can be determined by the self-adjoint criteria and have the following expressions

$$b_m = -\frac{2J_2\left(\frac{A_p R_y}{c}\right) + \frac{A_p R_y}{c} J_2'\left(\frac{A_p R_y}{c}\right)}{2Y_2\left(\frac{A_p R_y}{c}\right) + \frac{A_p R_y}{c} Y_2'\left(\frac{A_p R_y}{c}\right)}\quad (55)$$

$$E_q = -\frac{H_{\sqrt{5/2}k\pi}(a)}{{}_1F_1\left(-a, \frac{1}{2}, \frac{5k^2z^2}{2}\right) + 2a {}_1F_1\left(a + 1, \frac{3}{2}, -\frac{5}{2}k^2z^2\right)}\quad (56)$$

$$\begin{aligned}a = & \frac{-10k^2 + B_q^2 r_z^2(1 + \cosh(2k\pi))}{10k^2} \\ F_r = & -\frac{H_{\sqrt{5/2}k\pi}(b)}{{}_1F_1\left(-b, \frac{1}{2}, \frac{5k^2z^2}{2}\right) + 2b {}_1F_1\left(b + 1, \frac{3}{2}, -\frac{5}{2}k^2z^2\right)} \\ b = & \frac{-10k^2 + C_r^2 \Re_w^2(1 + \cosh(2k\pi))}{10k^2}.\end{aligned}\quad (57)$$

Hence our final expression for the action is given by

$$S = \frac{1}{2} \int d^4x \left[\sum_{pqr} \eta^{\mu\nu} \partial_\mu \phi_{pqr} \partial_\nu \phi_{pqr} + \sum_{abc pqr} M_{pqracb} \phi_{pqr} \phi_{abc} \right]\quad (58)$$

$$M_{pqracb} = \{A_p^2 \delta_{pa} \delta_{qb} \delta_{rc} + B_p^2 P_{rc} \delta_{pa} \delta_{qb} + C_p^2 P_{qb} Q_{rc} \delta_{pa} + m^2 P_{pa} Q_{qb} R_{rc}\},\quad (59)$$

where we have defined the following quantities,

$$P_{mn} = \int dy e^{-4cy} \alpha_n \alpha_m\quad (60)$$

$$Q_{mn} = \int dz \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \beta_n \beta_m\quad (61)$$

$$R_{mn} = \int dw \frac{\cosh^6(\ell w)}{\cosh^6(\ell\pi)} \gamma_n \gamma_m.\quad (62)$$

Now from the previous discussion we can find the solution for the three sets of functions $\alpha_n(y)$, $\beta_n(z)$ and $\gamma_n(w)$ which in turn determine P_{mn} , Q_{mn} and R_{mn} . Therefore from these three functions the explicit expression for KK mass modes can be determined from the quantity M_{pqracb} as given by Eq. (59). In this case as well the bulk field $\Phi(x, y, z)$ manifests itself to some four-dimensional observer as a scalar ϕ_{pqr} whose mass is determined by Eqs. (58) and (59).

The solution for $\alpha_n(y)$ as presented in Eq. (52) has the same nature as obtained by Wise *et al.* (see [13]). In this case as well lightest KK modes have mass modes determined by A_m , suppressed exponentially

with respect to the the scaler mass m which we have taken to be order of Planck scale. Thus these mass modes A_m are in the TeV range whereas m is of order of M_{pl} .

The solution for $\beta_n(z)$ and $\gamma_n(w)$ has been presented in Eqs. (53) and (54). The solution can be seen to include exponential factors such as $\exp[-\frac{5}{2}k^2z^2]$, $\exp[-3\ell^2w^2]$ and we see that when mass parameter B_m is of the order of TeV, solutions have maximum value around $z = 0$ as obtained earlier in Sec. III as well. From the solution of $\gamma_n(w)$ it is evident that the solution has maximum value around $w = 0$. Hence the bulk field being a product of these three functions $\alpha_p(y)$, $\beta_q(z)$ and $\gamma_r(w)$ as shown in Eq. (36), has mass parameter in the TeV range and has maximum value to find the modes around $(y = \pi, z = 0, w = 0)$ which is the location of the SM brane. Also the bulk field is maximum in the SM brane; i.e., the KK modes are most likely to be found in the TeV region as the A_m , B_m and C_m are in the TeV range. Along with the above line of arguments we could in principle have plotted all the functions $\alpha_p(y)$, $\beta_q(z)$ and $\gamma_r(w)$ and for all of them we have the functions to take maximum value at $y = \pi$, $z = 0$ and $w = 0$, precisely at the location of the TeV brane.

For completeness we present the self-interactions of the bulk scalar field in this seven-dimensional spacetime. From the four-dimensional point of view these self-interactions induce couplings between the KK modes. In this case also the effective self-couplings are suppressed by the warp factor and if the Planck scale sets the six-dimensional couplings and the low-lying KK modes have TeV range self-interactions. We present the interaction term in the action with coupling parameter λ such that

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \int_{-\pi}^{\pi} dw \sqrt{G} \frac{\lambda}{M^{5m-7}} \Phi^{2m}, \quad (63)$$

where the coupling λ is of the order of unity. Then we could expand in modes and hence the self-interactions of light KK states are given by

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \int_{-\pi}^{\pi} dw R_y r_z \mathcal{R}_w e^{-4\sigma} \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \frac{\cosh^6(\ell w)}{\cosh^6(\ell\pi)} \frac{\lambda}{M^{5m-7}} \phi_{pqr}^{2m} \left(\frac{\alpha_p}{\sqrt{R_y}} \frac{\beta_q}{\sqrt{r_z}} \frac{\gamma_r}{\sqrt{\mathcal{R}_w}} \right)^{2m} \quad (64)$$

The effective four-dimensional coupling constants therefore are being given by

$$\begin{aligned} \lambda_{eff} &= \frac{8\lambda}{(MR_y)^{m-1} (Mr_z)^{m-1} (M\mathcal{R}_w)^{m-1} M^{2m-4}} \int_0^\pi dy e^{-4\sigma} \alpha_p^{2m} \\ &\times \int_0^\pi dz \frac{\cosh^5(kz)}{\cosh^5(k\pi)} \beta_q^{2m} \int_0^\pi dw \frac{\cosh^5(\ell z)}{\cosh^5(\ell\pi)} \gamma_r^{2m}, \end{aligned} \quad (65)$$

which in the large kR_y , kr_z and $\ell\mathcal{R}_w$ limit reduces to

$$\begin{aligned} \lambda_{eff} &\simeq 8 \lambda \left(\frac{c}{MR_y} \right)^{m-1} \left(\frac{1}{Mr_z} \right)^{m-1} \left(\frac{1}{M\mathcal{R}_w} \right)^{m-1} \left(M e^{-c\pi} \frac{1}{\cosh^2 k\pi} \frac{1}{\cosh(\ell\pi)} \right)^{4-2m} \\ &\times \int_0^1 r^{4m-5} dr \left[\frac{J_2\left(\frac{A_p e^\sigma}{k} r\right)}{A_p} \right]^{2m} \int_0^\pi dz (\beta_q)^{2m} \int_0^\pi dw (\gamma_r)^{2m}. \end{aligned} \quad (66)$$

Hence we observe that the relevant scale for four-dimensional physics is not the scale set by Planck scale but $M e^{-c\pi} \frac{1}{\cosh^2 k\pi} \frac{1}{\cosh(\ell\pi)}$. Hence the KK reduction lead to the TeV scale couplings by the warp factor on the SM brane located at $(y = \pi, z = 0, w = 0)$.

Now this result can easily be extended to any higher dimension spacetime. For n extra dimensions we can write the action for the bulk field as,

$$S = \frac{1}{2} \int d^4x \int dy \int dz \int dw \cdots \sqrt{-G} [G_{AB} \partial^A \Phi \partial^B \Phi + m^2 \Phi^2] \quad (67)$$

where G_{AB} with $A, B = \mu, y, z, w, \cdots$ is given by a generalization of Eq. (41), and m is of order of M_{pl} . Then the KK splitting for the bulk field can be expressed as the following decomposition,

$$\Phi = \sum_{pqr\cdots} \phi_{pqr\cdots}(x) \frac{\alpha_p}{\sqrt{R_y}} \frac{\beta_q}{\sqrt{r_z}} \frac{\gamma_r}{\sqrt{\mathcal{R}_w}} \cdots \quad (68)$$

Thus among these n extra dimensions, one will have the solution given by Eq. (52), and then the other $(n - 1)$ solutions are being given by generalization of Eq. (53) such that the numerical values will be different but form of the solution remains unaltered. For n th extra dimension ($n > 1$) the solution for the mode can therefore be expressed as

$$\begin{aligned} \chi_r(w) = & \exp\left[-\frac{3}{2}k^2w^2\right] \exp\left[-\frac{1}{2}nk^2w^2\right] H_{\frac{\sqrt{3+n}}{2}kw}\left(\frac{-6k^2 - 2nk^2 + M_r^2r^2(1 + \cosh(2k\pi))}{2k^2(3+n)}\right) \\ & + F_r \exp\left[-\frac{3}{2}k^2w^2\right] \exp\left[-\frac{1}{2}nk^2w^2\right] \\ & \times {}_1F_1\left(-\frac{-6k^2 - 2nk^2 + M_r^2r^2(1 + \cosh(2k\pi))}{4k^2(3+n)}, \frac{1}{2}, \frac{1}{2}(3+n)k^2w^2\right) \end{aligned} \quad (69)$$

Hence the bulk field as viewed by a four-dimensional observer leads to a mass matrix whose components can be obtained by solving the eigenvalue problem as presented by each separable functions in the expansion given by Eq. (64). Also all these eigenvalues have TeV scale masses and the bulk field also has maximum value at $(y = \pi, z = 0, w = 0, \dots)$, which is the SM brane. Hence the standard Model particles can be taken as low-lying Kaluza-Klein modes of a bulk field propagating in any number of extra-dimensional spacetime.

The effective self-coupling in this case turns out to be

$$\begin{aligned} \lambda_{eff} \simeq 2^n \lambda \left(\frac{c}{MR_y}\right)^{m-1} \left(\frac{1}{Mr_z}\right)^{m-1} \left(\frac{1}{M\mathcal{R}_w}\right)^{m-1} \dots \left(Me^{-c\pi} \frac{1}{\cosh^{n-1} k\pi} \frac{1}{\cosh^{n-2}(\ell\pi)} \dots\right)^{4-2m} \\ \int_0^1 r^{4m-5} dr \left[\frac{J_2\left(\frac{A_p e^\sigma}{k} r\right)}{A_p}\right]^{2m} \int_0^\pi dz (\beta_q)^{2m} \int_0^\pi dw (\gamma_r)^{2m} \dots \end{aligned} \quad (70)$$

Thus finally we have obtained the KK mass modes and their self-interactions for n extra dimensions. We have also observed that in all these cases the KK mass modes and self-interactions are suppressed by the warp factor near the SM brane and hence all are in TeV scale. Hence this properties can be used to search for the TeV range KK mass modes and self-interactions in next generation colliders.

VI. DISCUSSION

In this paper we generalize the work presented in Ref. [13] on the bulk scalar field to determine its KK modes and the effective self-interaction in a multiple warped spacetime. For arbitrary number of extra dimensions, we have derived the expressions for the KK mode masses and their self-interactions. Various components determining the masses are in the TeV range because of the warp factor suppression. Moreover the bulk scalar field has been shown to have maximum value at the SM brane. Hence the low-lying KK modes for the bulk scalar fields lie in the TeV range with inverse TeV self-coupling. Thus the appearance of KK mode masses and couplings at TeV scale are generic features of warped dimensional models with any number of extra warped dimensions as long as we want to resolve the gauge hierarchy problem without introducing any hierarchical moduli. We have also introduced the moduli stabilization mechanism in these multiply warped models and have obtained the stabilized values for the moduli. Then we have presented a compact and generic formula to determine all the mass modes and their couplings for models with any arbitrary number of warped extra dimensions. This work now can be extended to other forms of bulk fields, which in turn may lead to the possibility of identifying various Standard Model particles as the low-lying KK excitation of various bulk fields, where the small warping in multiple directions can explain mass splitting in standard model particles as discussed in [17] and [18]. The close spacing of the low-lying KK modes along with enhanced coupling makes it likely for them to be seen as a series of close-lying resonances. In order to investigate the role of KK mass modes through collider-based experiments, we consider the interaction of various modes with themselves, i.e., self-interactions, and we have obtained that all of them are suppressed to the TeV scale by the warp factor. Also from the numerical values of masses for low-lying KK modes, we readily observe that the masses in the standard RS model get split into infinite number of mass modes, with very close spacings, which is a very interesting feature

of these multiply warped models and can be probed in future runs of LHC. The other things to be noted are that with stronger coupling of the KK modes of the bulk scalar field, one expects the decay widths to be larger, and thus the peaks to be broader as we go to higher and higher dimensions. The nature of the line shapes, therefore, will be an interesting benchmark to distinguish between higher-dimensional and lower-dimensional KK signals if such excitations appear during the high-luminosity runs of the LHC.

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